3. N. M. Belyanin, "Experimental investigation of friction and heat exchange for flow of a gas in a tube," Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 139-142 (1964).
4. A. I. Leont'ev and A. V. Fafurin, "Unsteady turbulent boundary layer in the initial section of the tube," Inzh.-Fiz. Zh., 25, No. 3, 389-402 (1973).
5. A. V. Fafurin and K. R. Shangareev, "Unsteady heat exchange under conditions of turning on and off a thermal load," in: Thermal Processes and Properties of the Working Substances in Aircraft Engines, Kazan Aviation Inst., No. 1 (1976), pp. 23-28.
6. A. I. Leont'ev, B. P. Mironov, and A. V. Fafurin, "Turbulent boundary layer of a dissociating gas in the initial section of the tube," Zh. Prikl. Mekh. Tekh. Fiz., No. 1, 100-105 (1967).

CLOSURE OF EQUATIONS OF TURBULENT FLOWS WITH TRANSVERSE SHEAR

A. P. Netyukhailo

UDC 532.526.3

Dependences are suggested for calculating rates of diffusion transport of kinetic turbulent energy and of scalar turbulence scale.

The description of turbulent effects by solving the exact Navier—Stokes equations encounters great difficulties at the present stage of development of fast computers. To solve practical engineering problems it is sufficient to calculate the average parameters of turbulent motion of a liquid and of heat and mass exchange. However, the equations of the averaged turbulent motion are not closed.

In calculating turbulent flows with account of its "prehistory," we use the system of equations:

$$\frac{\partial U_i}{\partial \tau} + U_j \frac{\partial U_i}{\partial x_j} = X_i - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial U_i}{\partial x_j} - \langle u_i u_j \rangle \right), \tag{1}$$

$$\frac{\partial U_i}{\partial x_i} = 0, \tag{2}$$

$$\frac{\partial E}{\partial \tau} + U_k \frac{\partial E}{\partial x_k} = - \frac{\partial}{\partial x_k} \left[\langle u_k E' \rangle + \left\langle \frac{p}{\rho} u_k \right\rangle \right] - \langle u_i u_k \rangle \frac{\partial U_i}{\partial x_k} - \nu \left\langle \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \frac{\partial u_i}{\partial x_k} \right\rangle, \tag{3}$$

in which the unknowns are the normal and tangential Reynolds stresses $\langle u_i u_j \rangle$, the rate of diffusion transport of kinetic energy turbulence $D_E = \frac{\partial}{\partial x_k} \left[\langle u_k E' \rangle + \left\langle \frac{p}{\rho} u_k \right\rangle \right]$, and the rate of its dissipation $D_E = \nu \left\langle \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \frac{\partial u_i}{\partial x_k} \right\rangle \simeq \left\langle \nu \left(\frac{\partial u_i}{\partial x_k} \right)^2 \right\rangle = \varepsilon.$

Analysis of the available closure models of Eq. (3) showed that approximating D_E by the form

$$D_E = \frac{\partial}{\partial x_k} \left(- \frac{\nu_т}{\sigma} \frac{\partial E}{\partial x_k} \right) \tag{4}$$

does not provide satisfactory agreement with experimental data [1, 2] (Fig. 1). This is explained by the fact that the available models do not take into account transport processes under the action of pressure pulsations [3]. Approximating the quantity ε by the Rotta equations [4] for $Re_E \gg 1$

VODGEO, Kharkov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 4, pp. 625-634, October, 1981. Original article submitted July 15, 1980.

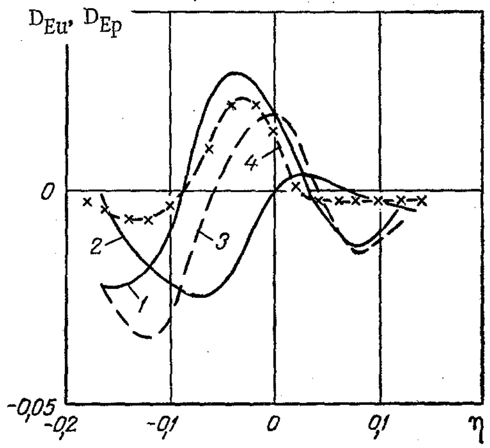

Fig. 1. Distribution of $\frac{\partial}{\partial x_k}\left[\langle u_k E'\rangle + \left\langle \frac{p}{\rho} u_k \right\rangle\right]$ over the mixing bed thickness [1]: 1) $D_{Eu} = \frac{\partial}{\partial x_k}\langle E' u_k\rangle$; 2) $D_{Ep} = \frac{\partial}{\partial x_k}\left\langle \frac{p}{\rho} u_k \right\rangle$; 3) $D_{Eu} + D_{Ep}$ (1-3, experimental data of [1]); 4) $\frac{\partial}{\partial x_3}\left(-\nu_T \frac{\partial E}{\partial x_3}\right)$.

$$\varepsilon = C_2 \frac{E\sqrt{E}}{l} \tag{5}$$

is found in agreement with numerous experimental data [1-4].

To approximate the correlation moment $\langle u_k E' \rangle$, we use the dynamic transport equation of triple central singular-point correlation moments of velocity pulsations [5, 6]:

$$\frac{D}{D\tau}\langle u_i u_j u_k\rangle + \langle u_i u_j u_r\rangle \frac{\partial U_k}{\partial x_r} + \langle u_j u_k u_r\rangle \frac{\partial U_i}{\partial x_r} + \langle u_k u_i u_r\rangle \frac{\partial U_j}{\partial x_r} - \langle u_i u_j\rangle \frac{\partial \langle u_k u_r\rangle}{\partial x_r} - \langle u_j u_k\rangle \frac{\partial \langle u_i u_r\rangle}{\partial x_r} - \langle u_i u_k\rangle \frac{\partial \langle u_j u_r\rangle}{\partial x_r} + \frac{\partial}{\partial x_r}\langle u_i u_j u_k u_r\rangle + \frac{1}{\rho}\left(\left\langle u_i u_j \frac{\partial p}{\partial x_k}\right\rangle + \left\langle u_j u_k \frac{\partial p}{\partial x_i}\right\rangle + \left\langle u_i u_k \frac{\partial p}{\partial x_j}\right\rangle\right) = 0. \tag{6}$$

Using the Poisson solution [6]:

$$p(x) = \frac{\rho}{4\pi}\iiint_V \left(\frac{\partial u_l}{\partial x'_m}\frac{\partial u'_m}{\partial x'_l} + \frac{\partial U'_l}{\partial x'_m}\frac{\partial u'_m}{\partial x'_l}\right)\frac{\partial V}{\mathbf{x}-\mathbf{x}'}, \tag{7}$$

we represent

$$\frac{1}{\rho}\left\langle u_i u_j \frac{\partial p}{\partial x_k}\right\rangle = \frac{\partial}{\partial x_k}\iiint_V \left[\left\langle \frac{\partial u'_l}{\partial x'_m}\frac{\partial u'_m}{\partial x'_l} u_i u_j\right\rangle + \frac{\partial U'_l}{\partial x'_m}\left\langle \frac{\partial u'_m}{\partial x'_l} u_i u_j\right\rangle\right]\frac{dV}{\mathbf{x}-\mathbf{x}'} - \iiint_V \left[\left\langle \frac{\partial u'_l}{\partial x'_m}\frac{\partial u'_m}{\partial x'_l}\frac{\partial (u_i u_j)}{\partial x_k}\right\rangle + \frac{\partial U'_l}{\partial x'_m}\left\langle \frac{\partial u'_m}{\partial x'_l}\frac{\partial \langle u_i u_j\rangle}{\partial x_k}\right\rangle\right]\frac{dV}{\mathbf{x}-\mathbf{x}'}. \tag{8}$$

Similar terms of Eq. (6) can be transformed by the same method. We approximate (8) in the form

$$\left\langle u_i u_j \frac{\partial p}{\partial x_k} \right\rangle \simeq \frac{\partial}{\partial x_k} \left(A_{ml}^{lmij} + \frac{\partial U_l}{\partial x_m} lB_l^{mij} \right) - \frac{1}{l} C_{mlk}^{lmij} - \frac{\partial U_l}{\partial x_m} D_{lk}^{mij} . \tag{9}$$

The tensors of expression (9) consist of a set of combinations (permutations) of correlation moments of velocity pulsations, unit symmetric and antisymmetric tensors. Below we provide for them final expressions with account of properties of unit tensors [7]:

$$A_{ml}^{lmij} \simeq A_1 \langle u_i u_j u_m^2 \rangle + A_2 E \langle u_i u_j \rangle , \tag{10}$$

$$B_l^{mij} \simeq B_1 \sqrt{E} \langle u_i u_j \rangle \delta_{lm} + B_2 E \sqrt{E} \delta_{ij} \delta_{lm} , \tag{11}$$

$$C_{mlk}^{lmij} \simeq G \sqrt{E} \langle u_i u_j u_k \rangle , \tag{12}$$

$$D_{lk}^{mij} \simeq D \langle u_m u_i u_j \rangle \delta_{lk} . \tag{13}$$

Keeping in mind that terms of Eq. (6) of type $\langle u_i u_j u_r \rangle \dfrac{\partial U_k}{\partial x_r}$ do not provide an important contribution to balance (6), and using the hypothesis of Millionshchikov [8]

$$\langle u_i u_j u_k' u_r' \rangle = \langle u_i u_j \rangle \langle u_k' u_r' \rangle + \langle u_i u_k' \rangle \langle u_j u_r' \rangle + \langle u_i u_r' \rangle \langle u_j u_k' \rangle$$

for the quasinormal probability distribution law, as well as relations (8), (9)-(13), we solve Eq. (6) for the correlation moment $\langle u_i u_j u_k \rangle$:

$$\langle u_i u_j u_k \rangle = - C_u \frac{l}{\sqrt{E}} \left(\langle u_k u_r \rangle \frac{\partial}{\partial x_r} \langle u_i u_j \rangle + \langle u_j u_r \rangle \frac{\partial}{\partial x_r} \langle u_i u_k \rangle + \langle u_i u_r \rangle \frac{\partial}{\partial x_r} \langle u_j u_k \rangle \right) -$$

$$- C_u' \frac{l}{\sqrt{E}} \left[\frac{\partial}{\partial x_i} (E \langle u_j u_k \rangle) + \frac{\partial}{\partial x_j} (E \langle u_i u_k \rangle) + \frac{\partial}{\partial x_k} (E \langle u_i u_j \rangle) \right] . \tag{14}$$

The right-hand side of relationship (14) coincides with the approximation of Hanjalic and Launder [6], and the second appears as a component in the Cormack multiparameter model [9]. Analysis of experimental data for flow in channels, boundary jets, free and boundary layers, as performed by Cormack, shows that for a suitable choice of constant coefficients the models of Hanjalic, Launder, and Cormack provide a satisfactory approximation to the measured values of $\langle u_i u_j u_k \rangle$ for any set of combinations of subscript values (i, j, k = 1, 2, 3).

Multiplying the instantaneous pressure value $\tilde{P}$ by $\tilde{U}_k$, using the Poisson solution, and averaging the result, we obtain

$$\tilde{P} \tilde{U}_k = PU_k + \langle pu_k \rangle = \frac{\rho}{4\pi} \iiint_V \left[U_k \frac{\partial U_m'}{\partial x_l'} \frac{\partial U_l'}{\partial x_m'} + \right.$$

$$\left. + U_k \left\langle \frac{\partial u_l'}{\partial x_m'} \frac{\partial u_m'}{\partial x_l'} \right\rangle + \left\langle u_k \frac{\partial u_l'}{\partial x_m'} \frac{\partial u_m'}{\partial x_l'} \right\rangle + \frac{\partial U_l'}{\partial x_m'} \left\langle U_k \frac{\partial u_m'}{\partial x_l'} \frac{\partial x_r}{\partial x_r} \right\rangle \right] \frac{dV}{|\mathbf{x} - \mathbf{x}'|} . \tag{15}$$

We approximate the pulsation part of expression (15) in the form

$$\left\langle \frac{p}{\rho} u_k \right\rangle \simeq U_k G_{lm}^{ml} + F_{ml}^{lmk} + \frac{\partial U_l}{\partial x_m} N_{lr}^{mkr} . \tag{16}$$

We provide the final expressions for the tensors appearing in (16):

$$G_{lm}^{ml} = G_1 E + G_2 \frac{\langle u_m u_l \rangle}{E} \langle u_m u_l \rangle ; \tag{17}$$

$$F_{ml}^{lmk} = F \langle E' u_k \rangle ; \tag{18}$$

$$N_{lr}^{mkr} = N \frac{l}{\sqrt{E}} \langle u_l u_m u_k \rangle . \quad (19)$$

Substituting (17)-(19) into (16), we obtain

$$\left\langle \frac{p}{\rho} u_k \right\rangle = C_G \left[U_k \left(E + C_G' \frac{(\langle u_m u_l \rangle)^2}{E} \right) \right] + C_F \langle u_k E' \rangle + C_N \frac{\partial U_l}{\partial x_m} \frac{l}{\sqrt{E}} \langle u_l u_m u_k \rangle . \quad (20)$$

The scalar turbulence scale appears in expressions (5), (14), and (20). To determine it we use the dynamic equation of dissipation rate transport of turbulent kinetic energy [5]:

$$\frac{\partial \varepsilon}{\partial \tau} + U_k \frac{\partial \varepsilon}{\partial x_k} = - \frac{\partial}{\partial x_k} (\langle \varepsilon' u_k \rangle) - 2 \frac{\nu}{\rho} \frac{\partial}{\partial x_i} \left\langle \frac{\partial p}{\partial x_l} \frac{\partial u_i}{\partial x_l} \right\rangle -$$

$$- 2\nu \frac{\partial U_i}{\partial x_k} \left\langle \frac{\partial u_i}{\partial x_l} \frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_i} \frac{\partial u_l}{\partial x_k} \right\rangle - 2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_l} \frac{\partial u_k}{\partial x_l} \right\rangle - 2 \left\langle \left(\nu \frac{\partial^2 u_i}{\partial x_k \partial x_l} \right)^2 \right\rangle . \quad (21)$$

We assume that the rate of diffusion transport of the quantity ε under the action of velocity pulsations depends on the average velocity shear and on the turbulence scale, i.e., ε is transported by large-scale vortices with velocity

$$W_k = G_\varepsilon \frac{\partial U_i}{\partial x_k} l, \quad (22)$$

and the diffusion flow ε is proportional to $W_k \varepsilon$.

The term of Eq. (21), taking into account the rate of diffusion transport ε under the action of pressure and velocity pulsations ε redistributed between components of velocity pulsations $2 \frac{\nu}{\rho} \frac{d}{dx_i} \left\langle \frac{dp}{dx_l} \frac{du_i}{dx_l} \right\rangle$, will be approximated on the basis of the transformed Navier–Stokes equations for $Re_E \gg 1$. Multiplying this term by $\partial \tilde{U}_i / \partial x_l$, differentiating the result obtained with respect to x_i, and then averaging, we obtain

$$-2 \frac{\nu}{\rho} \frac{\partial}{\partial x_i} \left\langle \frac{\partial p}{\partial x_l} \frac{\partial u_i}{\partial x_l} \right\rangle \simeq \frac{\partial U_l}{\partial x_k} \left\langle 2\nu \frac{\partial u_k}{\partial x_i} \frac{\partial u_i}{\partial x_l} \right\rangle +$$

$$+ \frac{\partial^2 U_l}{\partial x_i \partial x_k} \left\langle 2\nu u_k \frac{\partial u_i}{\partial x_l} \right\rangle + \frac{\partial}{\partial x_i} \left(U_k \left\langle 2\nu \frac{\partial u_l}{\partial x_k} \frac{\partial u_i}{\partial x_l} \right\rangle \right) +$$

$$+ \frac{\partial U_i}{\partial x_l} \left\langle 2\nu \frac{\partial u_k}{\partial x_i} \frac{\partial u_l}{\partial x_k} \right\rangle + \frac{\partial U_i}{\partial x_l} \left\langle 2\nu u_k \frac{\partial^2 u_l}{\partial x_i \partial x_k} \right\rangle +$$

$$+ \left\langle 2\nu \frac{\partial u_i}{\partial x_l} \frac{\partial u_k}{\partial x_i} \frac{\partial u_l}{\partial x_k} \right\rangle + \left\langle 2\nu u_k \frac{\partial u_i}{\partial x_l} \frac{\partial^2 u_l}{\partial x_i \partial x_k} \right\rangle . \quad (23)$$

In analyzing Eq. (23) we start from the condition that the turbulent structure of small scales is isotropic, since for the overwhelming majority of real nonisotropic turbulent flows the microstructure is isotropic (local isotropy). In this case the following condition can be adopted [7]:

$$\left\langle \left(\frac{\partial u_i}{\partial x_l} \right)_A \left(\frac{\partial u_k}{\partial x_m} \right)_B \right\rangle = - \frac{\partial^2}{\partial \xi_l \partial \xi_m} \langle u_i u_k \rangle \Big|_{A,B}$$

or

$$\left\langle \frac{\partial u_i}{\partial x_l} \frac{\partial u_k}{\partial x_m} \right\rangle = - \left[\frac{\partial^2}{\partial \xi_l \partial \xi_m} \langle u_i u_k \rangle \right]_{r=0} = - \langle u_i^2 \rangle \left(2\, \delta_{ik} \delta_{ml} - \frac{1}{2} \delta_{im} \delta_{kl} - \frac{1}{2} \delta_{il} \delta_{km} \right) \frac{\partial^2 f(r)}{\partial r^2} . \quad (24)$$

Here $\xi_l = (x_l)_B - (x_l)_A$, and f(r) is the mutual correlation coefficient between velocity pulsations at points A and B, being an even function of r. Its Taylor series expansion is

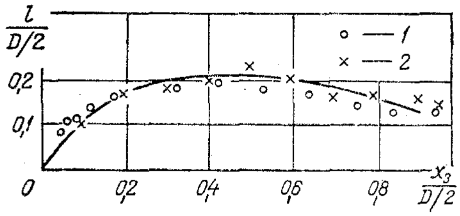

Fig. 2. Turbulence scale distribution over the channel width [10]: 1) by Eq. (42) [10]; 2) by relation (41).

$$f(r)=1+\frac{r^2}{2!}\left[\frac{\partial^2 f}{\partial r^2}\right]_{r=0}+\frac{r^4}{4!}\left[\frac{\partial^4 f}{\partial r^4}\right]_{r=0}+\ldots \tag{25}$$

By comparing the dependences

$$2\nu\left\langle\frac{\partial u_i}{\partial x_l}\,\frac{\partial u_i}{\partial x_l}\right\rangle=\varepsilon \tag{26}$$

and

$$\left\langle\frac{\partial u_i}{\partial x_l}\,\frac{\partial u_k}{\partial x_i}\right\rangle\simeq-2\langle u_i^2\rangle\left[\frac{\partial^2 f}{\partial r^2}\right]_{r=0} \tag{27}$$

it follows by account of (24) that

$$2\nu\left\langle\frac{\partial u_i}{\partial x_l}\,\frac{\partial u_k}{\partial x_i}\right\rangle\sim\varepsilon\delta_{lk}. \tag{28}$$

Dependence (28) makes it possible to estimate the terms of Eq. (23):

$$\frac{\partial U_l}{\partial x_k}\left\langle 2\nu\,\frac{\partial u_k}{\partial x_i}\,\frac{\partial u_i}{\partial x_l}\right\rangle\sim\frac{\partial U_l}{\partial x_k}\,\varepsilon\delta_{lk}=\frac{\partial U_l}{\partial x_l}\,\varepsilon, \tag{29}$$

$$\frac{\partial}{\partial x_i}\left(U_k\left\langle 2\nu\,\frac{\partial u_l}{\partial x_k}\,\frac{\partial u_i}{\partial x_l}\right\rangle\right)\sim\frac{\partial}{\partial x_k}\,(U_k\varepsilon), \tag{30}$$

$$\frac{\partial U_i}{\partial x_l}\left\langle 2\nu\,\frac{\partial u_k}{\partial x_i}\,\frac{\partial u_l}{\partial x_k}\right\rangle\sim\frac{\partial U_l}{\partial x_l}\,\varepsilon. \tag{31}$$

From the subscripts of the tensor $\frac{\partial^2 U_l}{\partial x_i\partial x_k}\left\langle 2\nu u_k\frac{\partial u_i}{\partial x_l}\right\rangle$ we infer its values by the conditions $\frac{\partial U_1}{\partial x_3}\gg\frac{\partial U_1}{\partial x_1},\ \frac{\partial U_3}{\partial x_3},\ \frac{\partial U_3}{\partial x_1}$:

$$\frac{\partial^2 U_l}{\partial x_i\partial x_k}\left\langle 2\nu u_k\,\frac{\partial u_i}{\partial x_l}\right\rangle\sim\frac{\partial^2 U_1}{\partial x_3^2}\left\langle 2\nu\,\frac{u_3}{\dfrac{\partial u_3}{\partial x_1}}\left(\frac{\partial u_3}{\partial x_1}\right)^2\right\rangle. \tag{32}$$

By dimensionality analysis of (32), we assume that there exist sizes of pulsations of linear vortex sizes, such that the following condition is satisfied

$$l'\sim\pm\frac{u_3}{\partial u_3/\partial x_1}. \tag{33}$$

For isotropic turbulence we take $\nu\left\langle\left(\frac{\partial u_3}{\partial x_1}\right)^2\right\rangle\sim\varepsilon.$ Then, assuming that the correlation

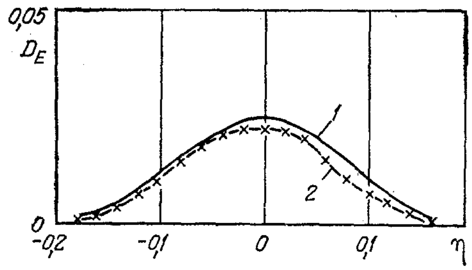

Fig. 3. D_E distribution over the mixing bed thickness [1]: 1) experimental data [1]; 2) by relations (40), (5).

coefficient between $\left\langle\left(\frac{\partial u_3}{\partial x_1}\right)^2\right\rangle$ and l' is a constant quantity, we rewrite (32) with account of (5)

$$\frac{\partial^2 U_l}{\partial x_i \partial x_k}\left\langle 2\nu u_k \frac{\partial u_i}{\partial x_l}\right\rangle \simeq C_{\varepsilon l} C_2 \left|\frac{\partial^2 U_1}{\partial x_3^2} E^{3/2}\right|. \tag{34}$$

Similar transformations are carried out over other terms of Eq. (23), using the well-known Kolmogorov hypothesis:

$$\nu_T = C_\tau \sqrt{E}\, l,$$

$$\frac{\partial U_i}{\partial x_l}\left\langle 2\nu u_k \frac{\partial^2 u_l}{\partial x_i \partial x_k}\right\rangle \simeq C_{\varepsilon l} C_2 \left|\frac{\partial U_1}{\partial x_3}\frac{\partial E^{3/2}}{\partial x_3}\right| + C_\tau C_{\varepsilon 1} C_2 \left(\frac{\partial U_1}{\partial x_3}\right)^2 E. \tag{35}$$

Based on this analysis of Eqs. (24)-(28), we reach the conclusion that

$$2\nu \frac{\partial U_i}{\partial x_k}\left\langle \frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right\rangle \sim \frac{\partial U_i}{\partial x_k}\delta_{ik}\varepsilon = 0, \tag{36}$$

since the relation $\frac{\partial U_i}{\partial x_i} = 0$ is valid for an incompressible liquid.

For approximating the remaining terms of Eq. (21), we use the analysis of [6]

$$2\nu\left\langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_l}\right\rangle + 2\left\langle\left(\nu \frac{\partial^2 u_i}{\partial x_k \partial x_l}\right)^2\right\rangle \simeq C_{\varepsilon\varepsilon}\frac{\varepsilon^2}{E}. \tag{37}$$

In analogy with (37), we write

$$2\nu\left\langle \frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right\rangle + 2\nu\left\langle u_k \frac{\partial u_i}{\partial x_l}\frac{\partial^2 u_l}{\partial x_i \partial x_k}\right\rangle \simeq F_\varepsilon \frac{\varepsilon^2}{E}. \tag{38}$$

Here F_ε can be a constant quantity or a function of the turbulence parameters. This problem will be investigated in what follows.

Based on the analysis of balance contributions of separate terms of Eq. (3) [1, 2], one can draw the conclusion that diffusion transport of turbulent energy under the action of pressure pulsations compensates to a large degree convective transport. We assume that the same compensation also occurs in transport of the quantity ε. With account of this assumption and approximations (29)-(32), (34)-(38) we solve Eq. (21) for the turbulence scale:

$$l = E\sqrt{(F_\varepsilon - C_{\varepsilon\varepsilon})C_2} \Bigg/ \left\{C_{\varepsilon 1} C_\tau \left(\frac{\partial U_1}{\partial x_3}\right)^2 E + C_{\varepsilon l}\left[\left|\frac{\partial^2 U_1}{\partial x_3^2} E^{3/2}\right| + \left|\frac{\partial U_1}{\partial x_3}\frac{\partial E^{3/2}}{\partial x_3}\right| + G\frac{\partial}{\partial x_3}\left(E^{3/2}\frac{\partial U_1}{\partial x_3}\right)\right]\right\}^{1/2} \tag{39}$$

The following dependence was obtained for boundary and free layers as a result of numerical optimization of coefficients

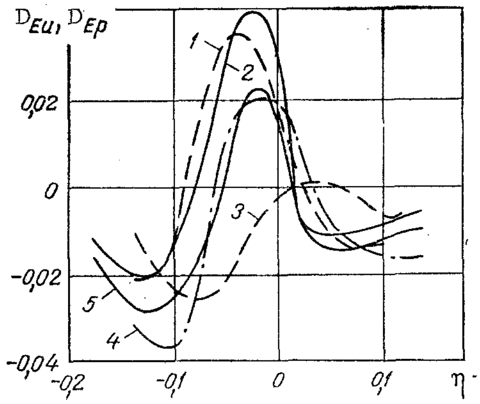

Fig. 4. Distributions of D_{Eu} and D_{Ep} over the mixing bed thickness [1]: 1) D_{Eu} from experimental data [1]: 2) D_{Eu} by relation (43); 3) D_{Ep} from experimental data [1]; 4) D_{Eu} + D_{Ep} from experimental data [1]; 5) by D_{Eu} + D_{Ep} (43) and (44).

$$l = 1.35\, E \Big/ \left[0.6\, E \left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left|E^{3/2} \frac{\partial^2 U_1}{\partial x_3^2}\right| + \left|\frac{\partial E^{3/2}}{\partial x_3} \frac{\partial U_1}{\partial x_3}\right|\right]^{1/2}. \tag{40}$$

The following formula is assumed in calculating turbulent characteristics in the channels

$$l = 1{,}75\, E \left(\sqrt{\frac{E}{E_{\max}}} - 0{,}075\right) \Big/ \left[0.6\, E \left(\frac{\partial U_1}{\partial x_3}\right)^2 + \left|E^{3/2} \frac{\partial^2 U_1}{\partial x_3^2}\right| + \left|\frac{\partial E^{3/2}}{\partial x_3} \frac{\partial U_1}{\partial x_3}\right|\right]^{1/2}. \tag{41}$$

Equations (40), (41) provide close results for flows in channels in the range $x_3/(D/2) = 0$–0.5 [10]. Figure 2 shows the distribution of the turbulence scale, calculated by Eq. (41) and

$$l = \int_0^\infty R_{11}(x_1,\ x_2,\ x_3 + r)\, dr. \tag{42}$$

The adequacy of Eq. (40) for free boundary layers is indicated by comparing the profiles $\varepsilon = \varepsilon(\eta)$ calculated by using Eq. (5) with the measured ones (Fig. 3). Significant deviations occur only in the outer boundary region, where significant errors are possible in determining derivatives with respect to velocity and energy. The calculated values of turbulence scales for flow in channels, boundary and free layers are in satisfactory agreement with measurements [11]. Using relations (14) and (20), and assuming that $\frac{d}{dx_3}\langle u_i^2\rangle \sim \frac{dE}{dx_3}$, we write in approximate form expressions for the rates of diffusion transport under the action of velocity and pressure pulsations in two-dimensional turbulent flows with transverse shear:

$$D_{Eu} = \frac{\partial}{\partial x_k}(\langle u_k E' \rangle) \simeq \frac{\partial}{\partial x_3}\left[-\sqrt{E}\, l \left(\frac{\langle u_3^2 \rangle}{E} + 0.1\right) \frac{\partial E}{\partial x_3}\right], \tag{43}$$

$$D_{Ep} = \frac{\partial}{\partial x_k}\left(\left\langle u_k \frac{p}{\rho}\right\rangle\right) \simeq \frac{\partial}{\partial x_3}\left[\sqrt{E}\, l \left(0{,}35 \frac{\langle u_3^2 \rangle}{E} + 0.035\right) \frac{\partial E}{\partial x_3}\right] - 0.5 \left\{\frac{\partial}{\partial x_1}[U_1 E(1 + f(R))] + \frac{\partial}{\partial x_3}[U_3 E(1 + f(R))]\right\}. \tag{44}$$

Here $f(R) = 0.405(1 + 0.4\, R^2)$, $R = lE^{-1}l^2 \partial U_1/\partial x_3$.

The proof of adequacy of models (43) and (44) was carried out for planar jets and mixing layers [1, 2], for which it is characteristic that the D_{Ep} distributions over the thickness of a layer with shear differ substantially from each other in magnitude and sign. The coefficient values of models (43), (44), obtained as a result of numerical optimization, provide satisfactory coincidence of $D_{Eu} + D_{Ep}$ values, measured [1, 2] and calculated by Eqs. (43) and (44) (Fig. 4).

Thus, as a result of the studies performed, a model equation was developed for describing E transport, each of whose terms separately agrees satisfactorily with its measured values [1, 2].

NOTATION

x_i, axes of the Cartesian coordinates (x_1, longitudinal direction; x_2, corresponds to the condition $\frac{dU_1}{dx_2}=0$; x_3, transverse direction); U_i, averaged velocity components; u_i, velocity pulsation components; p, pressure pulsation; $\tilde{U}_i$, $\tilde{P}$, instantaneous values of velocity and pressure; E, kinetic averaged turbulence energy; ε, kinetic turbulence energy dissipation rate; E', ε', turbulence energy and turbulence velocity pulsations; σ_{ij}, ε_{ijk}, single symmetric and antisymmetric tensors; ν_T, turbulent viscosity coefficient; ν, kinematic molecular viscosity coefficient; ρ, density; V, volume; l, turbulence scale; A^{mlij}_{lr}, sixth rank tensor; x, x', radii-vectors for the position in space of two points at a distance of r apart; $R_{11}(x_1, x_2, x_3 + r)$, coefficient of intercorrelation of longitudinal velocity pulsations; η, dimensionless transverse coordinate; P, averaged pressure.

LITERATURE CITED

1. I. Wygnanski and H. E. Fiedler, "The two-dimensional mixing region," J. Fluid Mech., 42, 327-363 (1970).
2. G. Kheskestad, "Thermoanemometer measurements in a planar turbulent jet," Prikl. Mekh., 32, No. 4, 1-17 (1965).
3. W. C. Reynolds, "Recent advances in the computation of turbulent flows," Adv. Chem. Eng., 9, 193-246 (1974).
4. J. Rotta, "Statistical theory of inhomogeneous turbulence," Z. Phys., B129, 547-572; B131, 51-77 (1951).
5. B. I. Davydov, "Statistical dynamics of an incompressible turbulent fluid," Dokl. Akad. Nauk SSSR, 136, No. 1, 47-50 (1961).
6. K. Hanjalic and B. E. Launder, "A Reynolds stress model of turbulence and its application to thin shear flow," J. Fluid Mech., 52, 609-639 (1972).
7. J. O. Hinze, Turbulence, McGraw-Hill, New York (1975).
8. M. D. Millionshchikov, "The theory of homogeneous isotropic turbulence," Dokl. Akad. Nauk SSSR, 32, 615-619 (1941).
9. D. E. Cormack, L. G. Leal, and J. H. Seinfeld, "An evaluation of mean Reynolds stress turbulence models: the triple correlation velocity," Trans. ASME, 100, 47-54 (1978).
10. J. Conte-Bello, Turbulent Flow in a Channel with Parallel Walls [Russian translation], Mir, Moscow (1968).
11. A. P. Netjukhajlo, I. A. Sherenkov, and O. L. Tertichny, "The model of turbulent exchange in stratified flows," Proc. XVIII Congress IAHR, Italy, Vol. 3, 109-116 (1979).